Integrazione di tecniche di ricerca operativa e constraint programming per ottimizzazione multiobiettivo

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Constraint Optimization Problem

- A COP a CSP (X,D,C) with a cost function *f* to minimize

$$f: D_1 \times \ldots \times D_N \rightarrow S_t$$

  where $(S_t, \leq)$ is a total order
- An assignment *A* is an optimal solution to a COP iff it is a solution of the CSP and $\neg\exists A'$ s.t. $f(A') < f(A)$
- Total order among solutions:
  - Only the *best* assignment satisfying constraints is considered solution of the COP

Multi-criteria Optimization Problems

- A MOP is a CSP (X,D,C) with functions

  $f \equiv f_1, f_2, \ldots, f_n$ that *"should be optimized at the same time"*

- The user *is not able* to synthesize the functions into only one
  - usually, *tradeoff* solutions are considered more interesting, *extreme* solutions are seldom accepted
- In most cases there is not only *one* optimal point

Non-Dominated Frontier

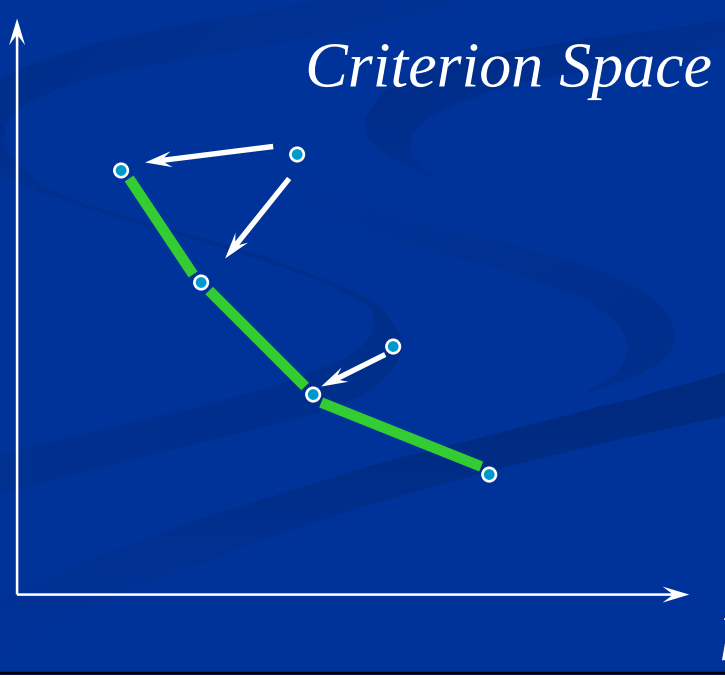
- In a MOP, the concept of *better solution* turns into the concept of *Domination*:

$$\underline{X} \leq_d \underline{Y} \Leftrightarrow \forall\ k=1..n,\ X_k \leq Y_k$$

- A Solution of the CSP is *Pareto-Optimal* or *Non-Dominated* iff

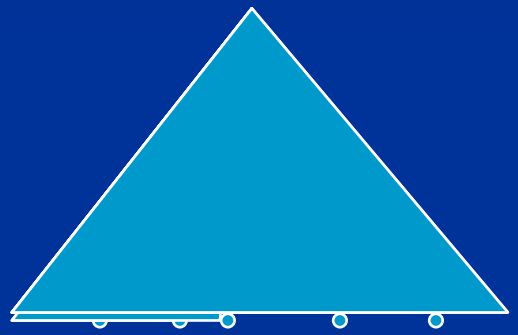
  $\neg\exists A'$ s.t. $\underline{f(A')} <_d \underline{f(A)}$

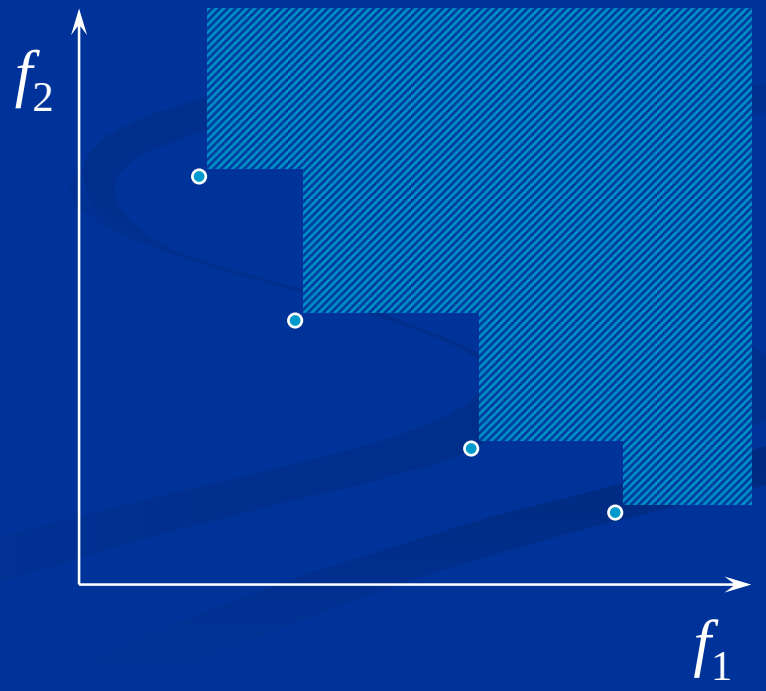
- Only points in the nondominated frontier are interesting to the user



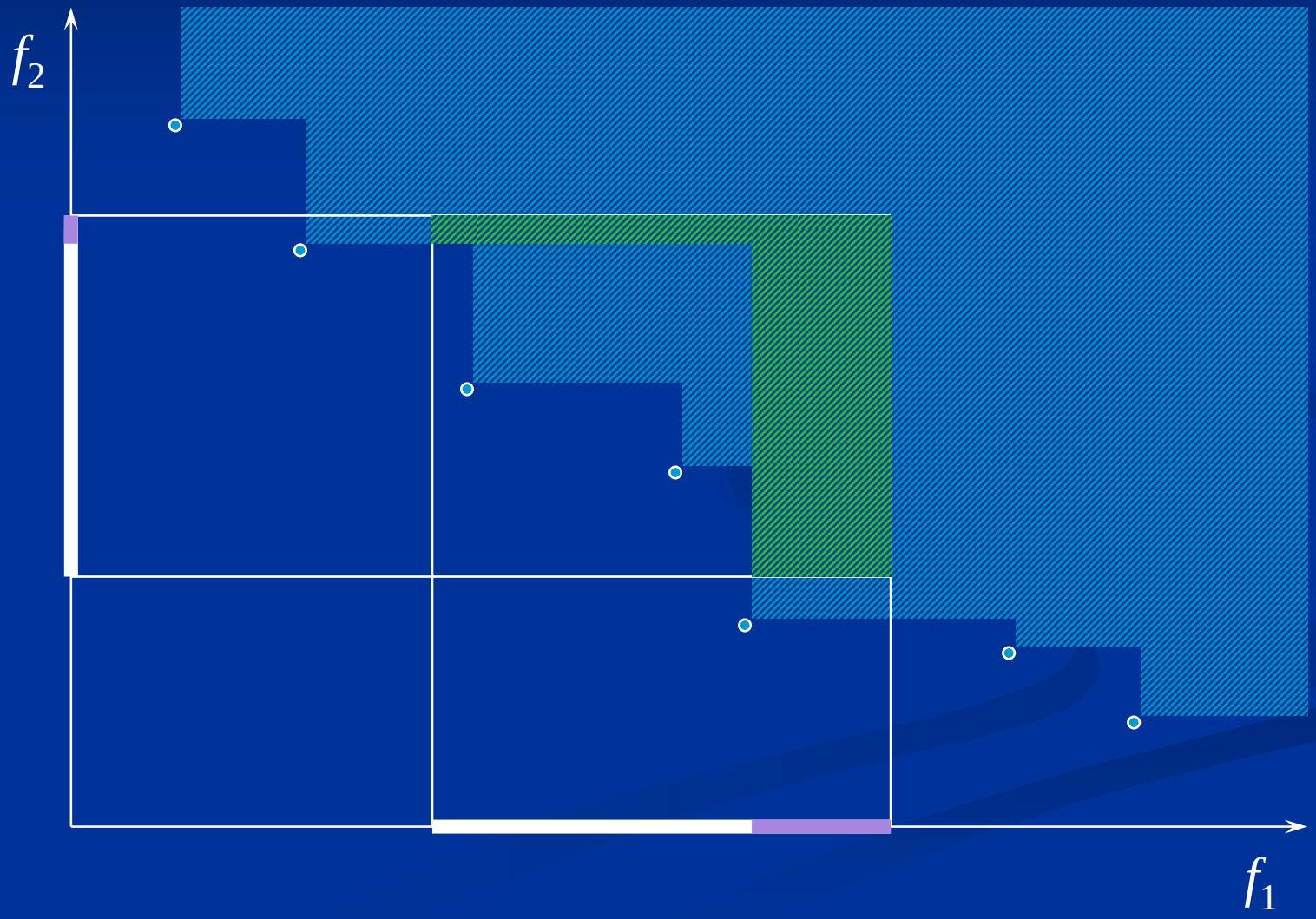
PCOP-B&B

Search Tree





Propagation of unbacktrackable constraints

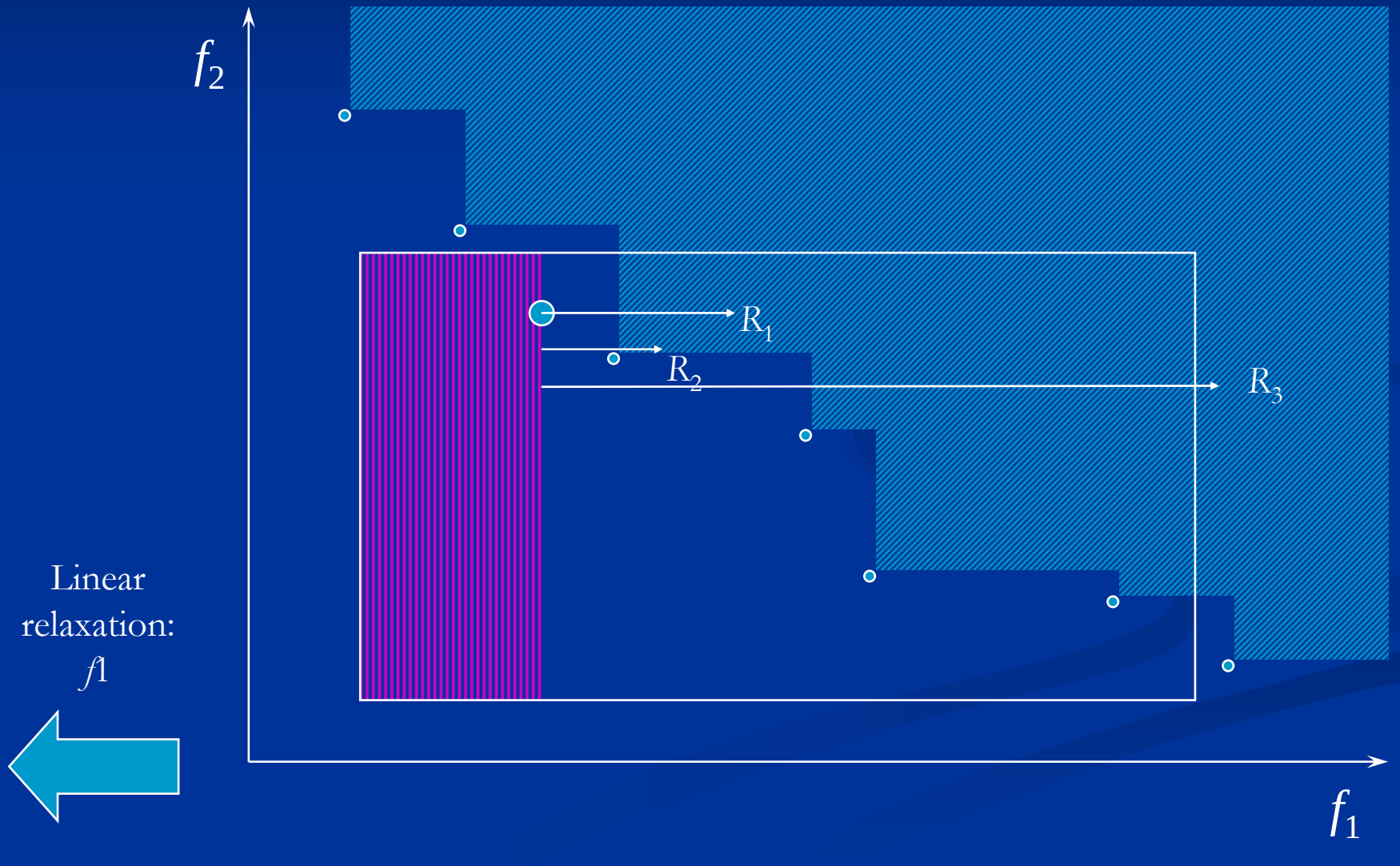


Integration with OR

- Linear relaxation of the problem
  - provides a better bound on the objective function value (e.g., function f_1)
  - provides reduced costs
    - for variable X_i, the reduced cost R_i gives

      how much the objective function will worsen, if we add 1 unit of variable X_i in solution

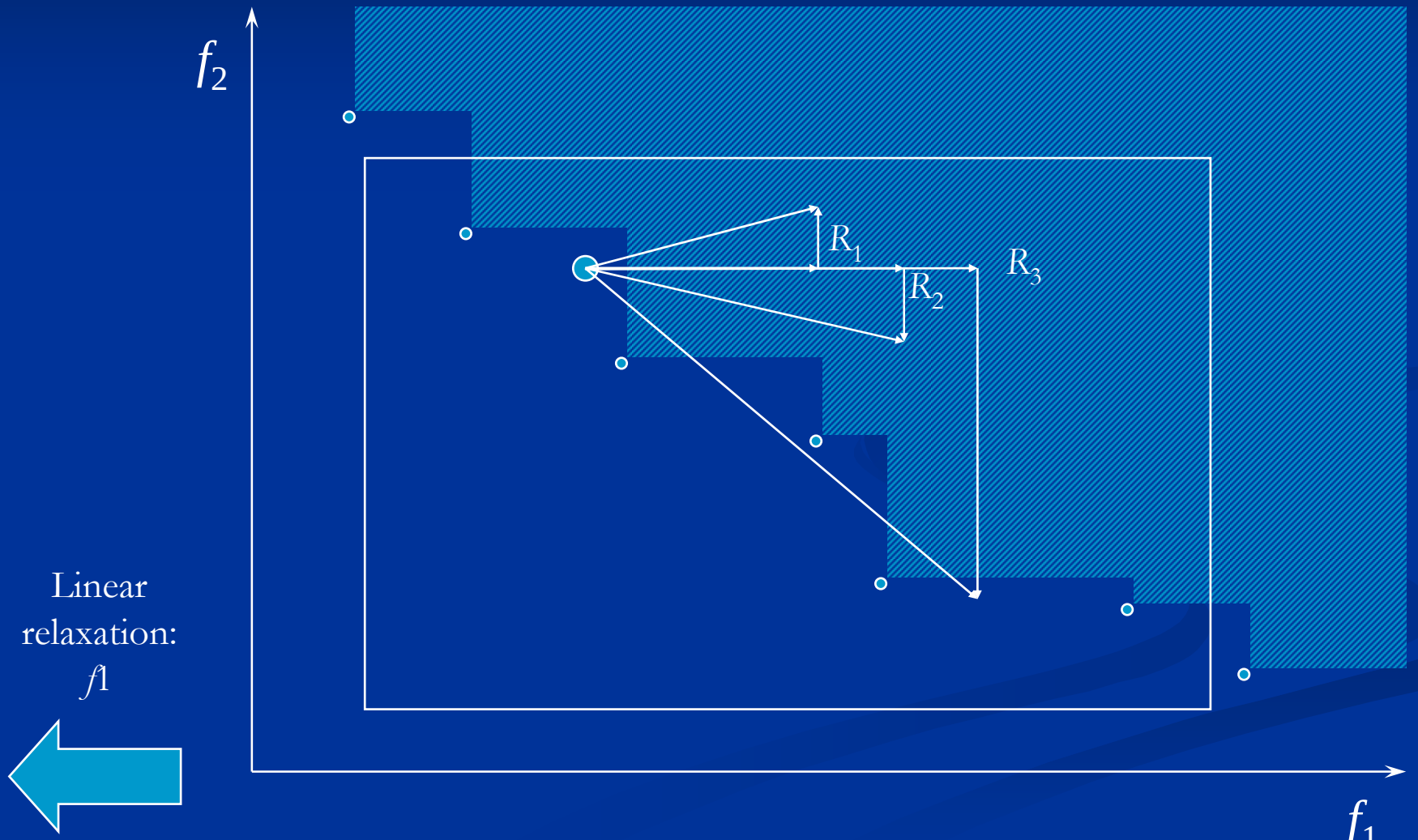
Classic Propagation of bounds & reduced costs



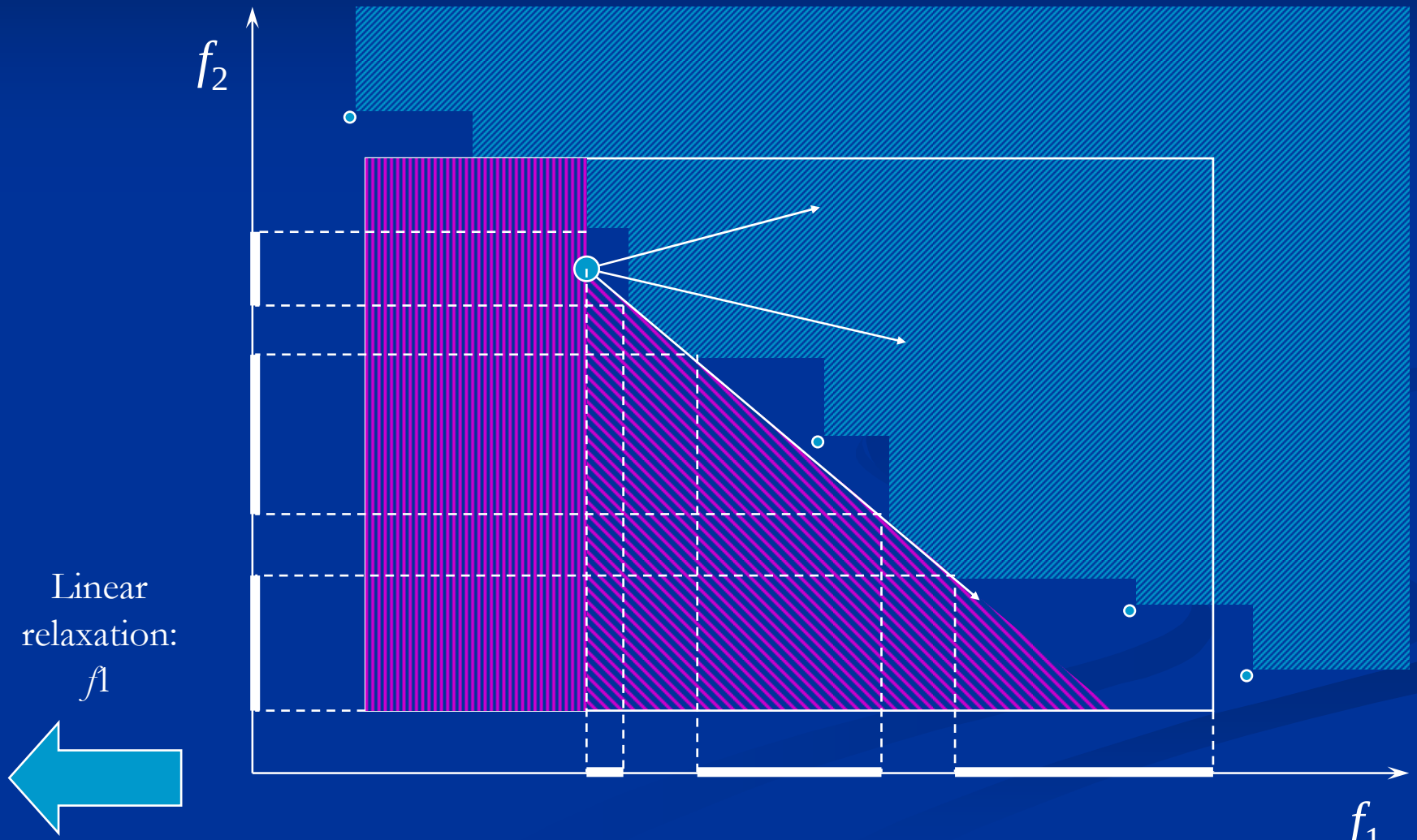
Reduced costs wrt f_2

- We can obtain the reduced costs wrt other objective functions
- They can be positive or negative
- If they are all positive
  - the solution is optimal also wrt *f2*
  - otherwise, they show how the solution changes wrt f2
- The criterion space of the linear relaxation is convex

Reduced costs wrt both functions



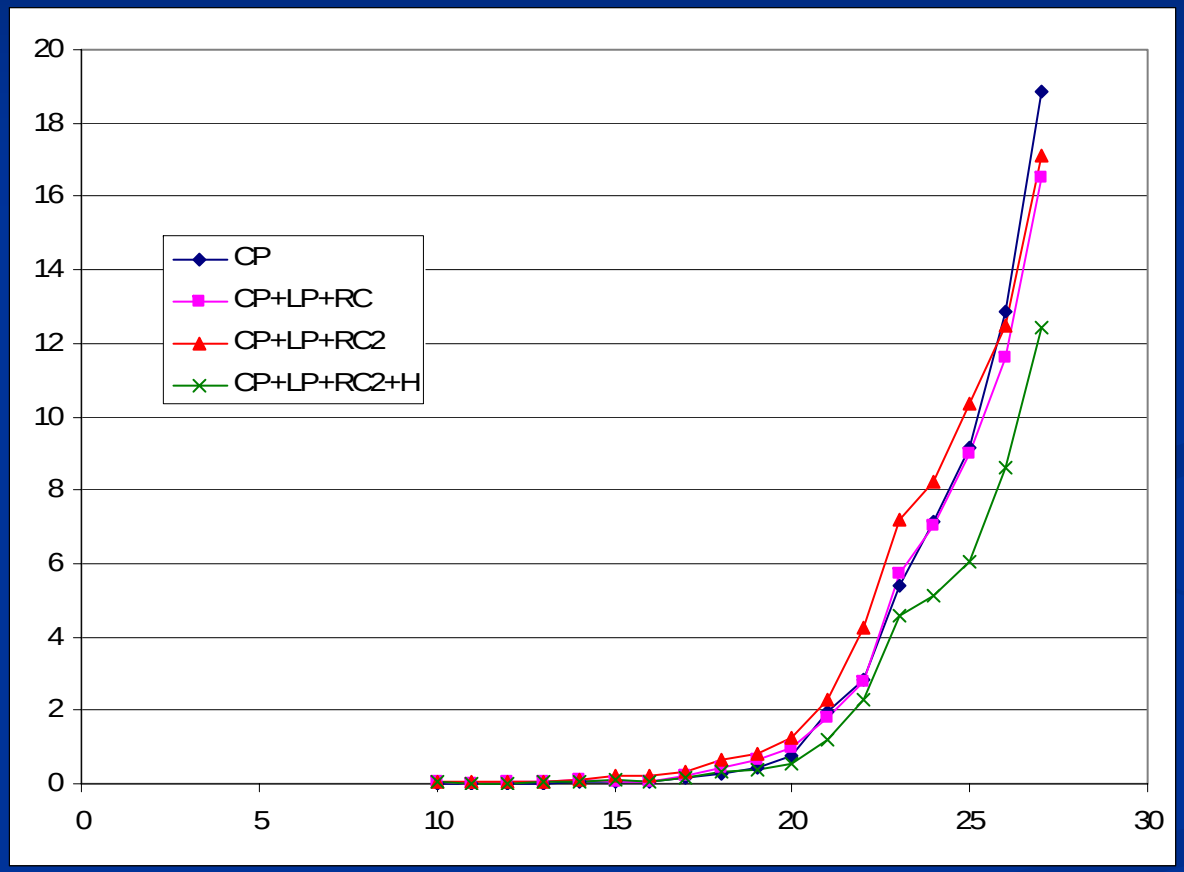
Reduced costs wrt both functions



Heuristics based on Reduced costs

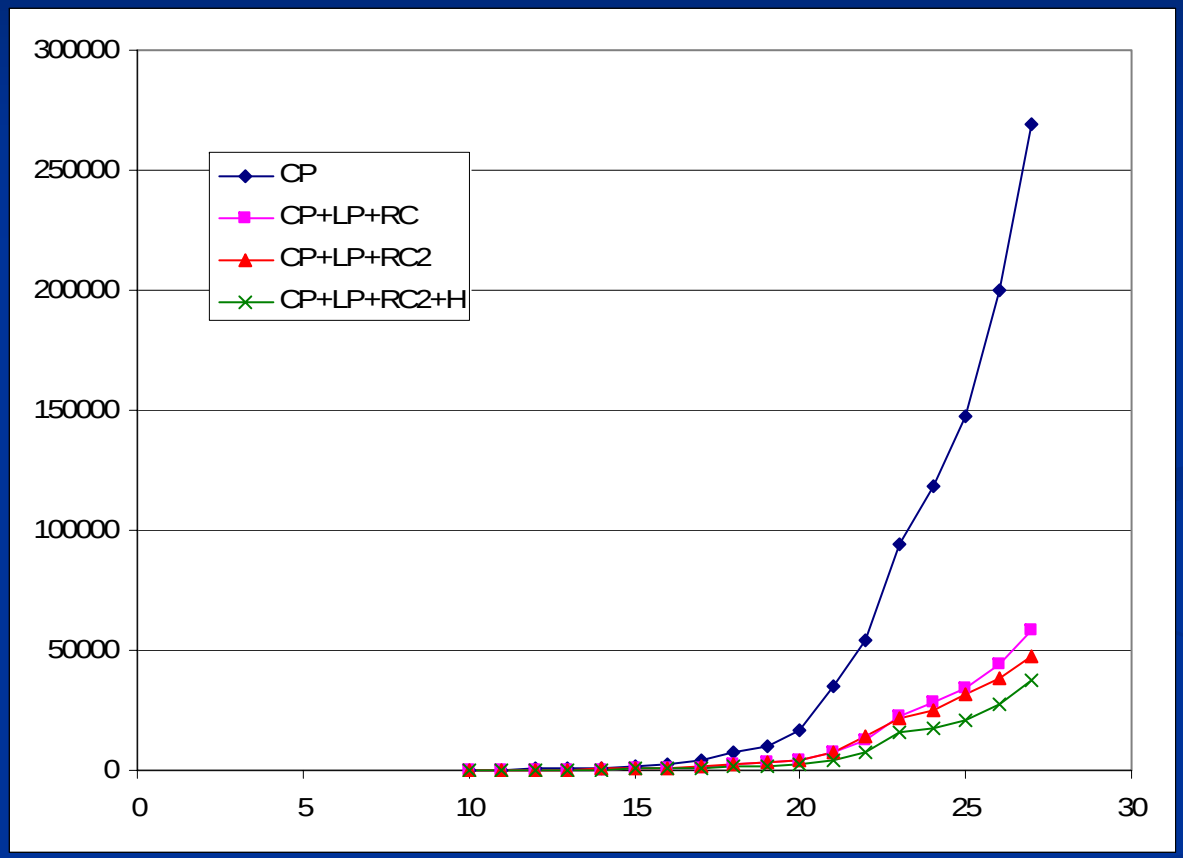
- If one variable has reduced costs of the same sign → the two objective functions agree on the value that variable should take
- Heuristics: assign this value in early nodes of the search tree

Preliminary results



- Multi-knapsack
- timing results
- ECLiPSe + XpressMP

Preliminary results



- Backtracks

Future work

- Experiment with problems better suited for the integration CP+LP
- Other uses of reduced costs (heuristics with LDS)
- Use of the dual solution for detecting optimality
- Sensitivity analysis
- Integrate CP with local search, genetic algorithms
- Implement in other systems (ILOG+CPLEX)?