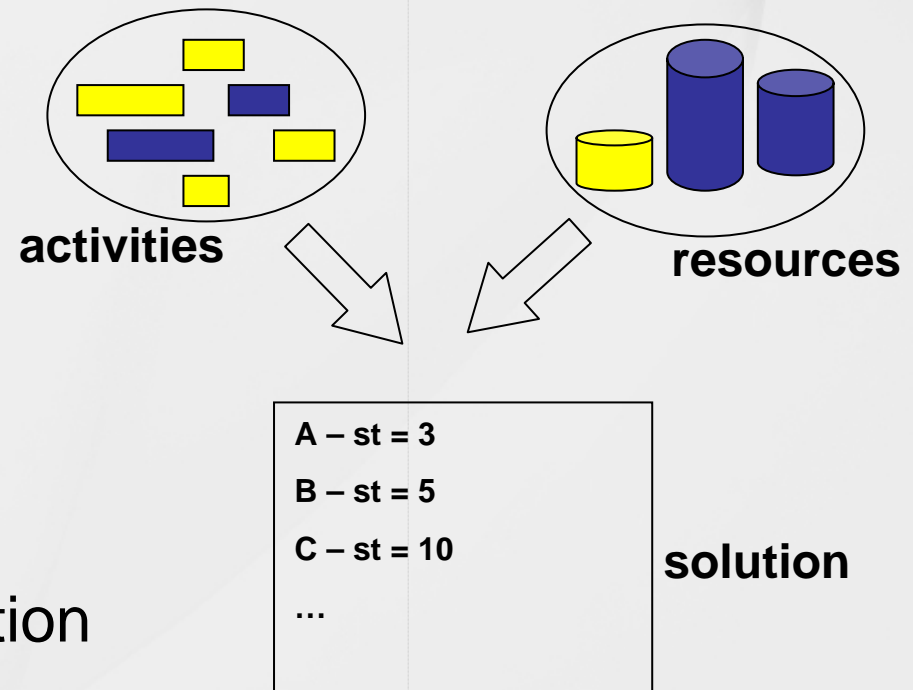


Scheduling with Uncertainty

Nicola Policella

Scheduling Problem

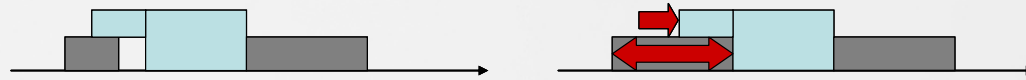
- Scheduling is the problem of assigning a set of actions/tasks to a set of resources (machines, workman, trucks) subject to a set of constraints.
- A set of activities
- A set of resources
- A set of constraints
 - Temporal
 - Resource
 - Causal
- One or more objective function
- Solution: define the start time for each activity



Scheduling with Uncertainty

- Different scheduling techniques have been developed to analyze scheduling problems and produce high quality solutions
- Unfortunately, in the real world there is a degree of uncertainty
 - Activities can last longer
 - Arrival of new activities
 - Resource capacity variations

Scheduling with Uncertainty



temporal uncertainty

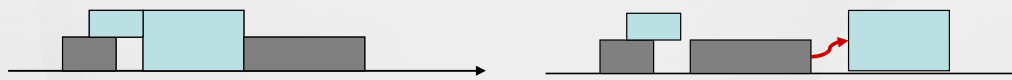
Activities last longer than expected or they can be postponed



resource uncertainty

Difference between nominal (left) and actual (right) resource availability.

Reduction of resource availability blocks the execution of some activities and their consequent delay



causal uncertainty

A new precedence relation between a pair of activities requires a revision of previous choices

Real world

- In real working environments, unforeseen events tend to quickly invalidate solution
 - Schedule's life is short!
- Approaching a scheduling problem requires the coupling of:
 1. a **predictive** scheduling engine, to propose a possible solution
 2. a **reactive** scheduling engine, to manage the current solution and to make “repairs” during the execution

Examples in Current Research Scenario

predictive

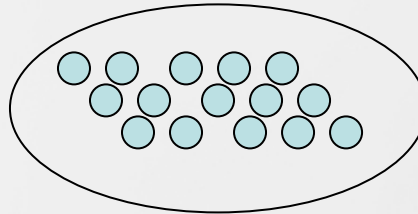
- Producing solutions based on uncertainty knowledge
 - Synthesis of robust solutions [Davenport et al., 01]
- Using flexibility to cope with uncertainty
 - Partially defined solutions [Wu et al., 99]

reactive

- Dealing with uncertainty during schedules execution
 - Local versus Global approach [Smith, 94; El Sakkout & Wallace, 2000]

Our predictive approach

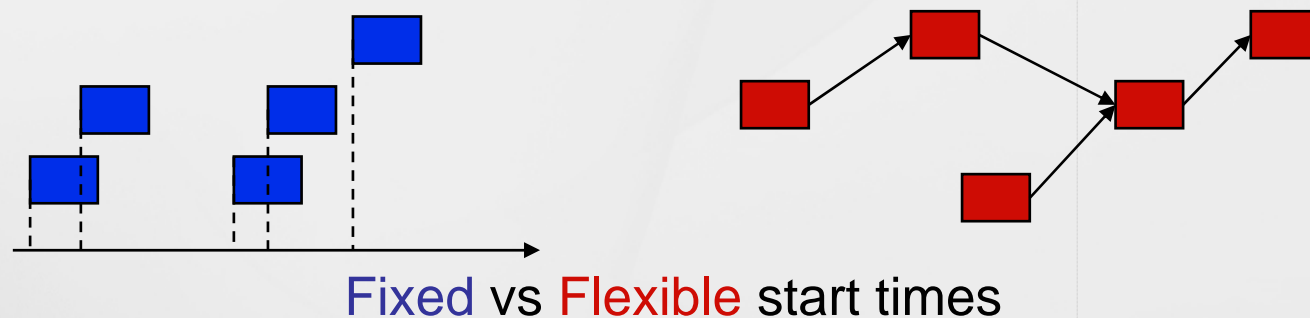
- Many scheduling solutions instead of 1!
 - Many possible start times for each activity



- A set of solutions to face different world evolutions
- To avoid, during the execution phase, a new search step to select a new solution in this set
 - A means to “extract” a new solution in a fast way

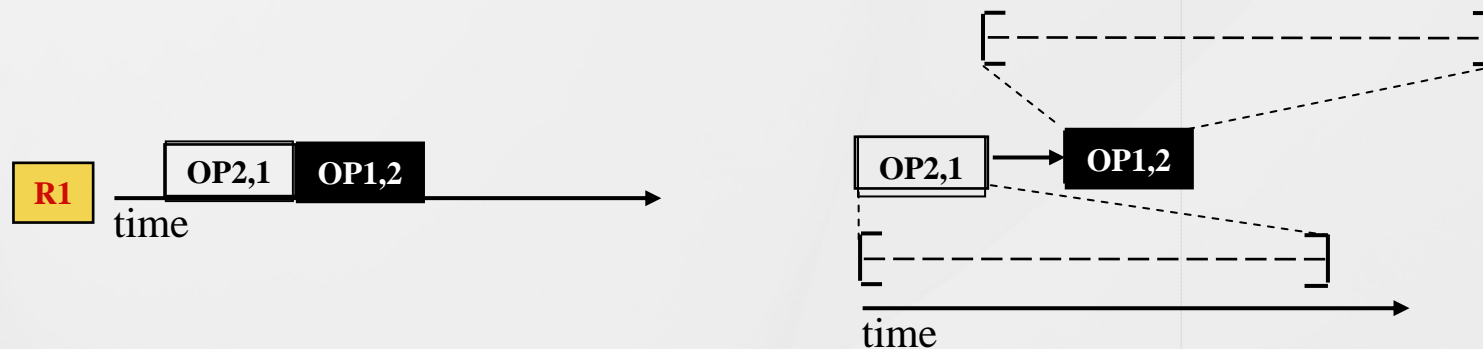
Our Predictive Approach

- Two aspects of robustness are investigated
 - the ability of a solution to absorb external events
 - the ability to keep pace with execution
- Focus on temporal flexibility
 - Synthesize a set of possible schedules
 - Have an associated method to change current solution
- Basic idea of constructing a **Partial Order Schedule** (POS) instead of a fixed time schedule



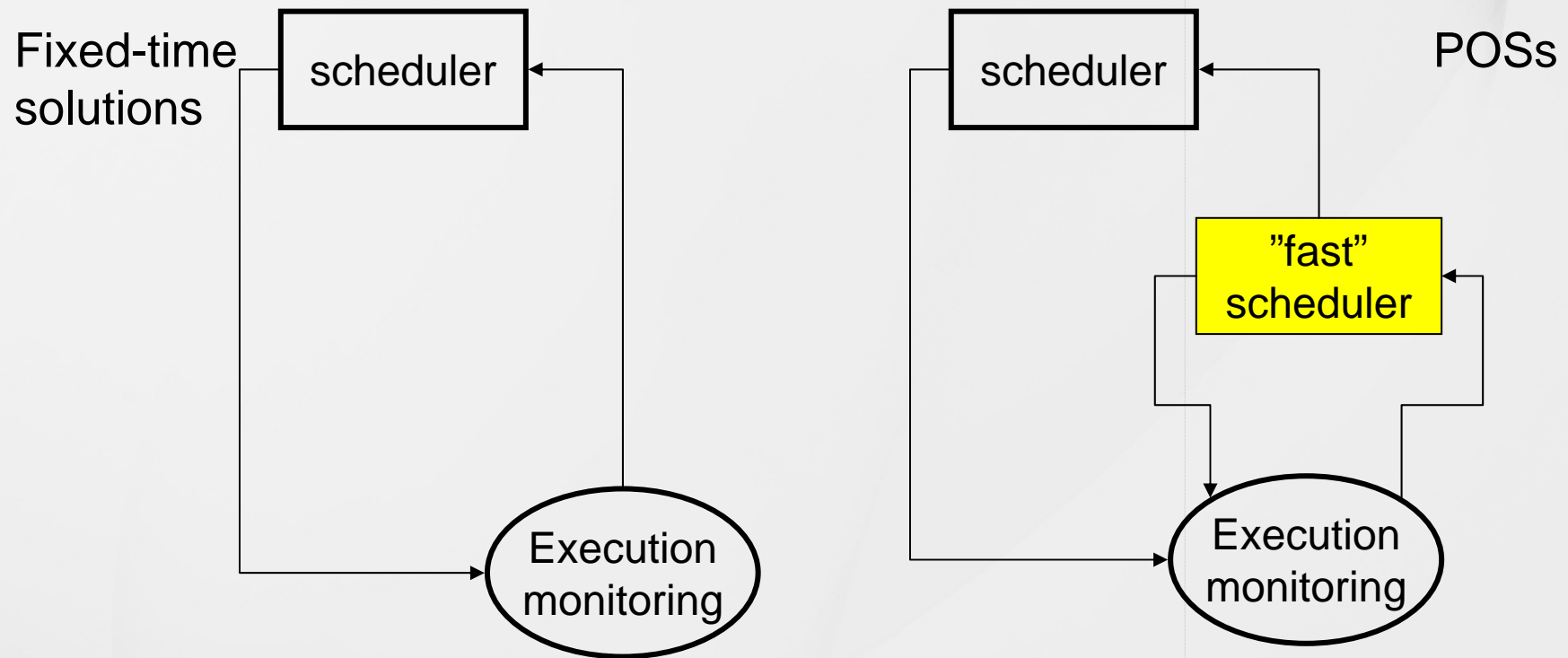
Partial order schedule

- Given a scheduling problem a Partial Order Schedule (POS) is a set of solutions that can be represented as a temporal graph [any time feasible schedule defined in the graph is also a resource feasible schedule]
- An interesting property of POSs is “fast re-scheduling”:
 - many external changes may have a reactive response accomplished via simple propagation in the underlying temporal network (a polynomial time calculation).

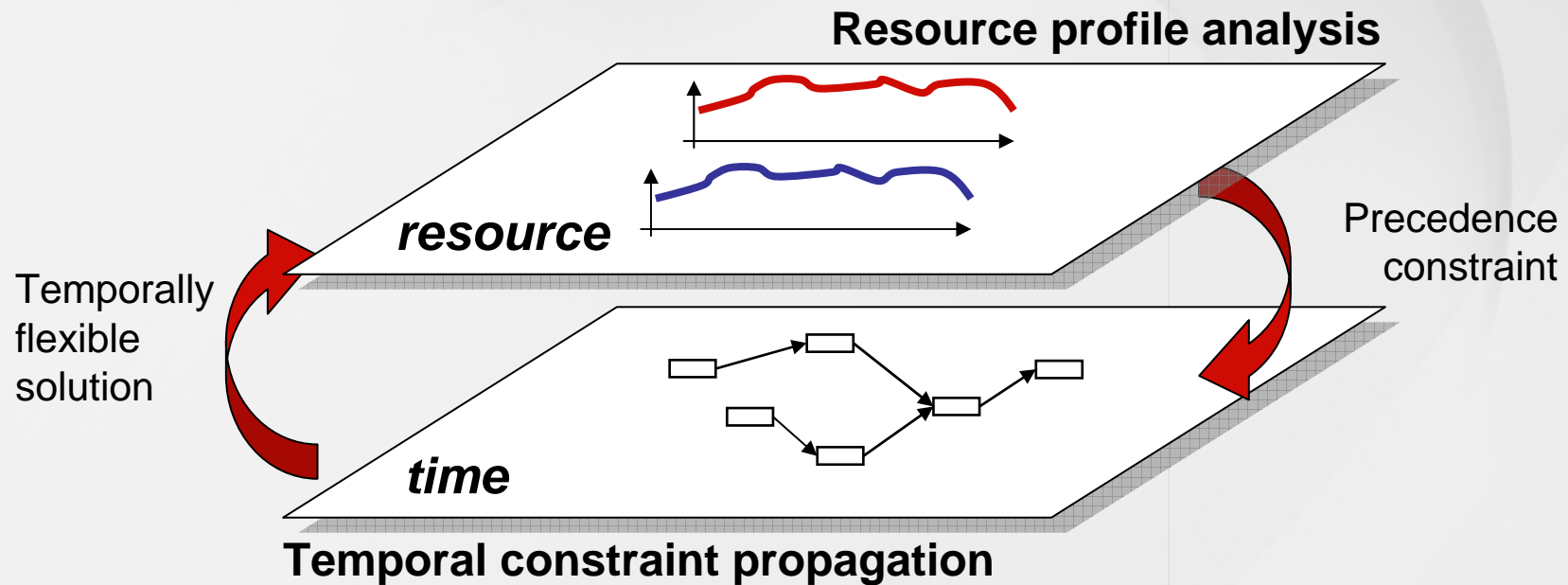


- Sequence activities that compete for resources letting start and end times float

Partial order schedule



How to compute POSs



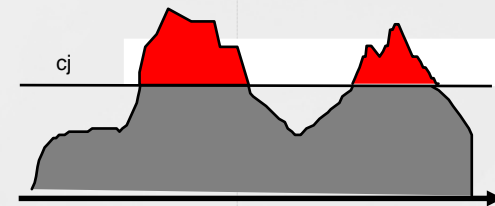
- Variables:
 - start and end times of each activity
- Domains
 - A schedule horizon $[0, H]$
- Constraints
 - Temporal constraints (e.g., duration of activities, min-max separation between activities, simple precedences)
 - Resource constraints (e.g., bounds on capacities)

Precedence Constraint Posting

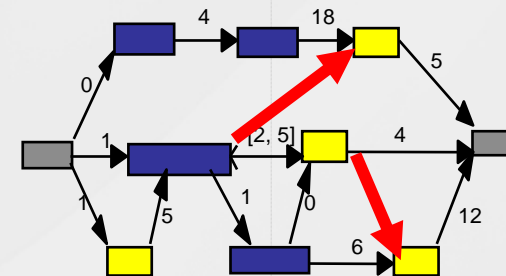
```
loop
  propagate(CSP)
  compute-conflicts(CSP)
  if no-conflict then
    return-solution
  else
    if unresolvable-conflicts then
      return-fail
    else
      select-conflict
      select-precedence
      post(precedence)
end-loop
```

Remove resource violations *posting*
further precedence constraint in the
temporal network

SOLUTION FOUND!



Resource profile

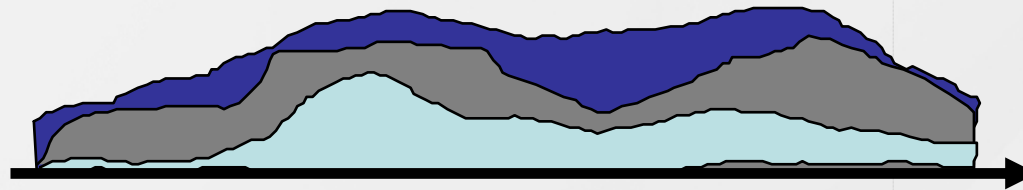


Temporal Net

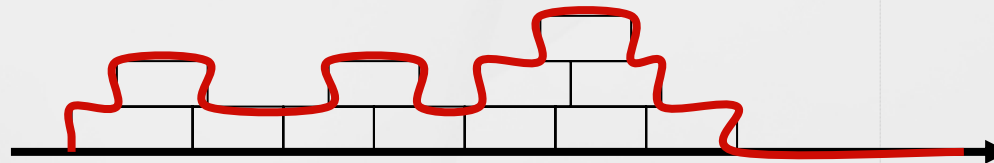
Inconsistent values can be pruned in
polynomial time (Dechter, 91)

Two different resource profiles

- Because of the flexibility of the temporal network an exact computation of demand profile is not possible
- An intuitive compromise consists of computing its upper-bound and lower-bound projections

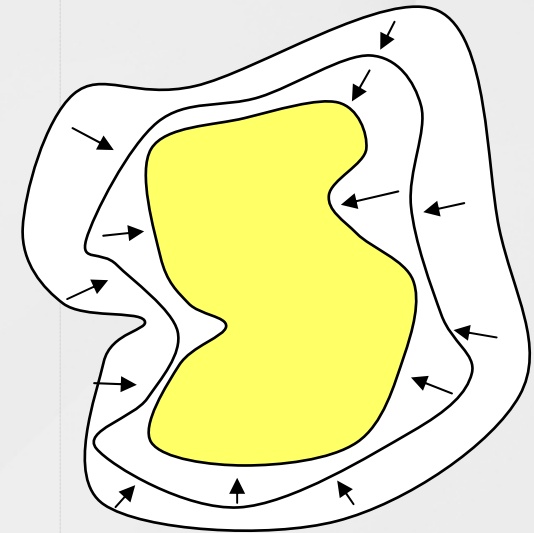


- A different way is to compute the resource profile for a specific point in the search space:
 - Earliest start time solution



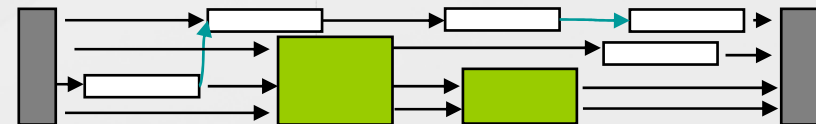
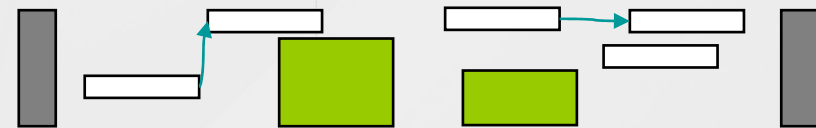
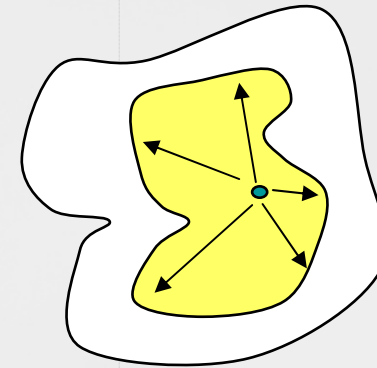
EBA: Envelope Based Analysis

- A least commitment POS generation
- Maintain the flexibility of temporal network
- Compute the tightest possible resource bounds of the “flexible” profile using Resource Envelope computation [Muscettola, 02]

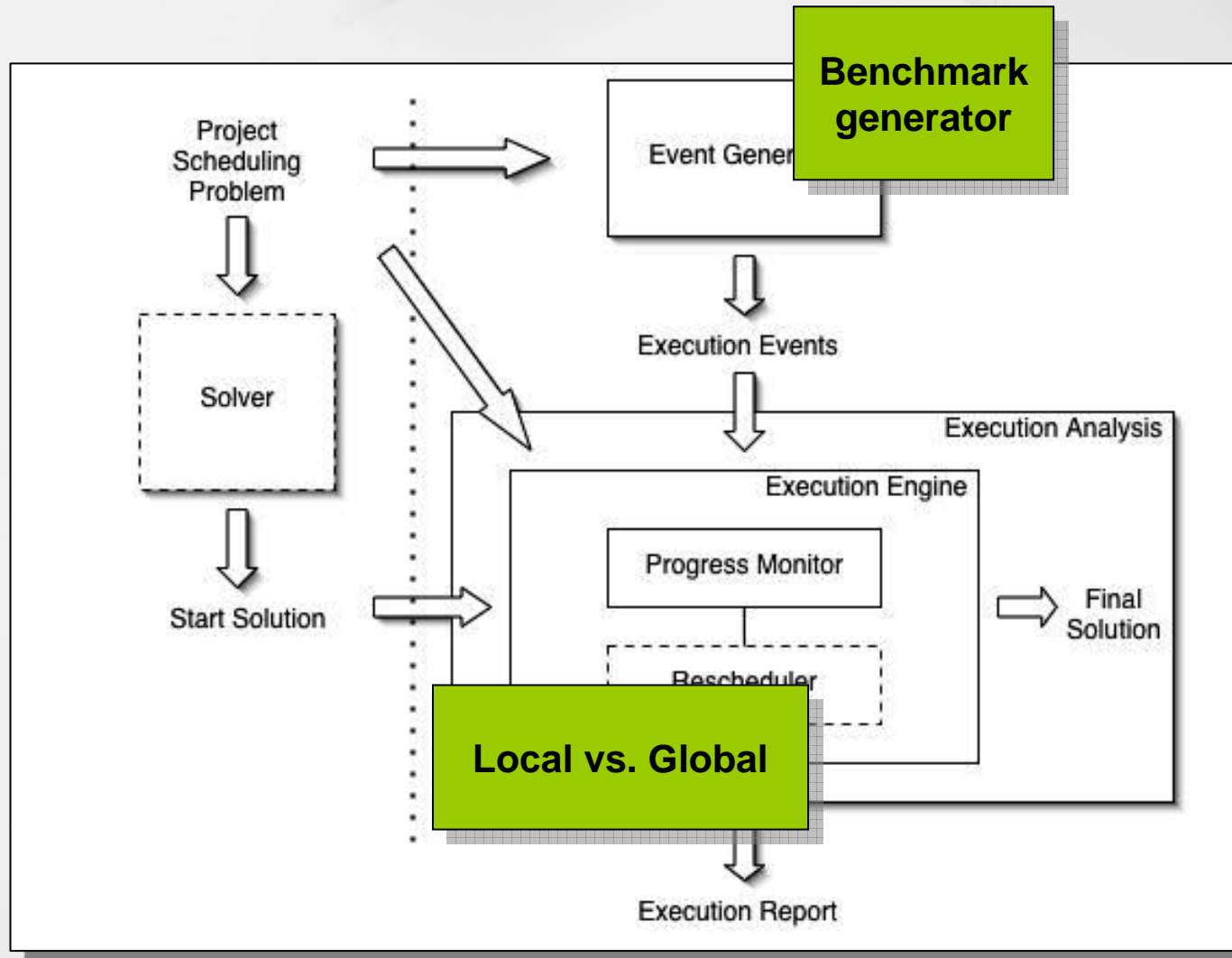


ESTA^C: Earliest Start Time Analysis

- Computing a Resource Profile in the Earliest Start Time of the temporal network
- Two Step process
 - Generate a single solution reasoning on the earliest start time profile
 - Post process the solution transforming it into a POS through chaining



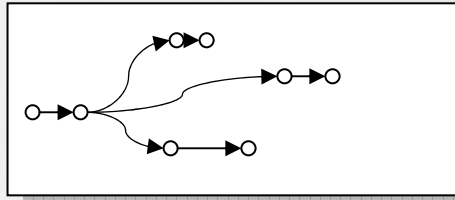
- Simulation of POS execution
 - Comparison of POSs with other types of scheduling solution
 - Fixed time solutions
 - Flexible solutions
 - Identifying possible drawbacks in POSs
 - Analyzing reactive methods to manage POS execution



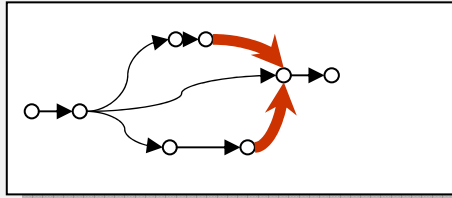
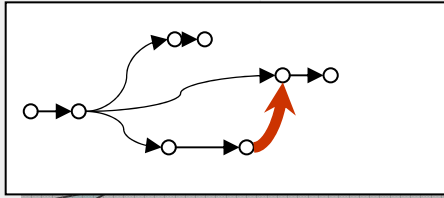
An architecture to schedule execution and monitoring

Off-line

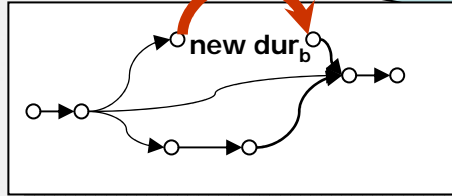
Produce a **partial order schedule**



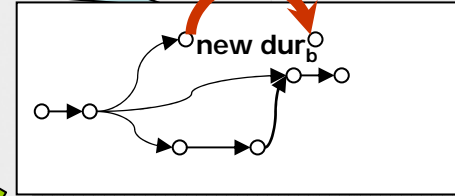
Produce a **flexible schedule**



unexpected event!
*Activity **b** lasts more than expected*



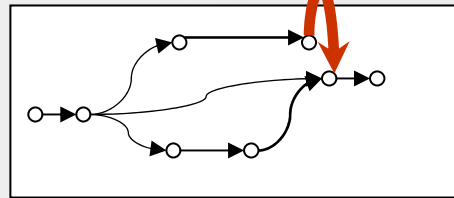
New solution: POS does not need any rescheduling



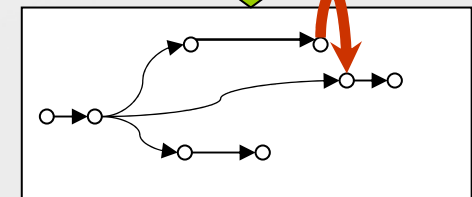
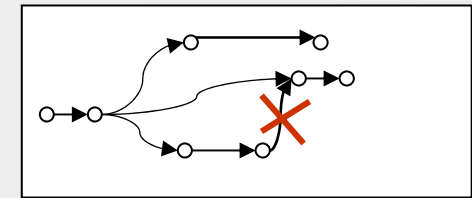
Two rescheduling strategies

no-retraction

retraction



New solution



On-line

First results and ongoing work

- In the case of temporal changes we have:
 - POSs assure a fast and stable answer
 - POSs present a, not so predictable, scarce capability in accepting exogenous events
- The evaluation suggests several research lines
 - the production of a different POSs
 - the introduction of different reactive techniques.