Uncertainty in Bipolar Preference Problems

Stefano Bistarelli[1,2], Maria Silvia Pini[3], Francesca Rossi[3], and K. Brent Venable[3]

1 : Dipartimento di Scienze, Università "G. d'Annunzio", Pescara, Italy
2 : Istituto di Informatica e Telematica, CNR, Pisa, Italy
E-mail: Stefano.Bistarelli@iit.cnr.it
3 : Dipartimento di Matematica Pura ed Applicata, Università di Padova, Italy
E-mail: {mpini,frossi,kvenable}@math.unipd.it

Abstract. Preferences and uncertainty are common in many real-life problems. In this paper, we focus on bipolar preferences and on uncertainty modelled via uncontrollable variables. However, some information is provided for such variables, in the form of possibility distributions over their domains. To tackle such problems, we eliminate the uncertain part of the problem, making sure that some desirable properties hold about the robustness of the problem's solutions and its relationship with their preference. We also define semantics to order the solutions according to different attitudes with respect to the notions of preference and robustness.

1 Introduction

Bipolar preferences and uncertainty are present in many application fields, such as satellite scheduling, logistics, and production planning. For example, in multi-agent problems, agents may express their preferences in a bipolar way, and variables may be under the control of different agents. To give a specific example, just consider a conference reviewing system, where usually preferences are expressed in a bipolar scale. Uncertainty can arise for the number of available conference rooms at the time of the acceptance decision, and the goal could be to select the best papers while ensuring that they all can be presented. In general, in many real-life situations agents express what they like and what they dislike, thus often preferences are bipolar.

In this paper, bipolarity is handled via the formalism in [3]. Other formalisms can be found in [8, 1, 4, 5]. We choose to generalize to bipolar preferences the soft constraints formalism [2] which is able to model problems with one kind of preferences (i.e., the negative preferences). Thus, each partial instantiation within a constraint will be associated to either a positive or a negative preference.

Another important feature, which arises in many real world problems, is uncertainty. We model uncertainty by the presence of *uncontrollable* variables. This means that the value of such variables will not be decided by us, but by Nature or by some other agent. Thus a solution will not be an assignment to all the variables but only to the controllable ones. A typical example of uncontrollable variable, in the context of satellite scheduling, is a variable representing the time when clouds will disappear. Although we cannot choose the value for such uncontrollable variables, we have some information on the plausibility of the values in their domains. In [7] this information, which is not bipolar, is given by probability distributions. In this paper, we model this information by a possibility distribution over the values in the domains of such variables. Possibilities are useful when probability distributions are not available [11].

After defining formally bipolar preference problems with possibilistic uncertainty, we define the notion of preference and robustness for the solutions of such problems, as well as properties that they should respect, also in relation to the solution ordering. We then concentrate on problems with totally ordered preferences defined over real intervals, and we show how to eliminate the uncontrollable part of the problem by adding new constraints on the controllable part to recall part of the removed information. This approach is a generalization of the approach used in [9] to remove uncertainty from problems with negative preferences only. The additional constraints are then considered to define the robustness of the solutions. We define formally the preference and robustness of the solutions, and we define some desirable properties related to such notions that the solution ordering should have. Moreover, we introduce semantics that use such notions to order the solutions, and we show that they satisfy the desired properties on the solution ordering. In particular, they allow us to distinguish between highly preferred solutions which are not robust, and robust but not preferred solutions. Also, they guarantee that, if there are two solutions with the same robustness (resp., the same preference), then the ordering is given by their preference (resp., robustness).

2 Background: bipolar preference problems

Bipolar preference problems [3] are based on a bipolar preference structure, which allows to handle both positive and negative preferences. This structure contains two substructures, one for each kind of preferences.

When dealing with negative preferences, two main properties should hold: combination should bring to worse preferences, and indifference should be better than all the other negative preferences. These properties can be found in a c-semiring [2], which is the structure used to represent soft constraints. A *c-semiring* is a tuple $(A, +, \times, \mathbf{0}, \mathbf{1})$ where: A is a set, $\mathbf{0}, \mathbf{1} \in A$, $+$ and $\times$ are the additive and the combination operators. Operator $+$ induce a partial order, written $\leq_S$, over A: $a \leq_S b$ iff $a+b = b$, $\mathbf{0}$ is its minimum and $\mathbf{1}$ its maximum. When $a \leq_S b$, we will say that *b is better than a*. Element $\mathbf{1}$ acts as indifference (in fact, $\forall a \in A$, $a \times \mathbf{1} = a$), and $\forall a, b \in A$, $a \times b \leq a, b$. This interpretation is natural when considering the weighted c-semiring $(R^+, min, +, +\infty, 0)$, where preferences are real positive numbers interpreted as costs, and thus as negative preferences. Such costs are combined via the sum $(+)$ and the best costs are the lower ones (min). From now on, a c-semiring will be denoted as: $(N, +_n, \times_n, \bot_n, \top_n)$.

When dealing with positive preferences, combination should bring to better preferences, and indifference should be lower than all the other positive preferences. These properties can be found in a *positive preference structure* [3], that is a tuple $(P, +_p, \times_p,$ $\bot_p, \top_p)$, which is just like a negative one above, except that the combination operator $\times_p$ returns a better element rather than a worse one. An example of a positive preference structure is ($\Re^+$,max,sum,0,$+\infty$), where preferences are positive real numbers aggregated with sum and ordered bymax (i.e., the best preferences are the highest ones).

When we deal with both positive and negative preferences, the same properties described above for a single kind of preferences should continue to hold. Moreover, all the positive preferences must be better than all the negative ones and there should exist an operator allowing for the compensation between positive and negative preferences. A bipolar preference structure links a negative and a positive structure by setting the highest negative preference to coincide with the lowest positive preference to model indifference. More precisely, a *bipolar preference structure* [3] is a tuple (N, P, $+$, $\times$, $\bot$, $\Box$, $\top$) where, (P, $+_{|P}$, $\times_{|P}$, $\Box$, $\top$) is a positive preference structure; (N, $+_{|N}$, $\times_{|N}$, $\bot$, $\Box$) is a c-semiring; $+ : (N \cup P)^2 \longrightarrow (N \cup P)$ is an operator s.t. $a_n + a_p = a_p$, $\forall a_n \in N$ and $a_p \in P$; it induces a partial ordering on $N \cup P$: $\forall a, b \in P \cup N$, $a \leq b$ iff $a + b = b$; $\times : (N \cup P)^2 \longrightarrow (N \cup P)$ (called the *compensation operator*) is a commutative and monotonic operator. In the following, we will write $+_n$ instead of $+_{|N}$ and $+_p$ instead of $+_{|P}$. Similarly for $\times_n$ and $\times_p$. When $\times$ is applied to a pair in ($N \times P$), we will sometimes write $\times_{np}$. An example of bipolar structure is the tuple (N=$[-1, 0]$, P=$[0, 1]$, $+$=max, $\times$, $\bot$=-1, $\Box$=0, $\top$=1), where $\times$ is s.t. $\times_p$= max, $\times_n$=min and $\times_{np}$=sum. Negative preferences are between -1 and 0, positive preferences between 0 and 1, compensation is sum, and the order is given by max.

A bipolar constraint is a constraint where each assignment of values to its variables is associated to one of the elements in a bipolar preference structure. A *bipolar CSP* (BCSP) $\langle S, V, C\rangle$ is a set of bipolar constraints C over a set of variables V defined on the bipolar structure S.

We will sometimes need to distinguish between two kinds of constraints in a BCSP. For this reason, we will use the notion of *RBCSP*, which is a tuple $\langle S, V, C_1, C_2\rangle$ such that $\langle S, V, C_1 \cup C_2\rangle$ is a BCSP.

Given a subset of variables $I \subseteq V$, and a bipolar constraint $c = \langle def, con\rangle$, the *projection of* c over I, written $c \Downarrow_I$, is a new bipolar constraint $\langle def', con'\rangle$, where $con' = con \cap I$ and $def(t') = \sum_{\{t|t\downarrow_{con'}=t'\}} def(t)$. In particular, the preference associated to each assignment to the variables in con', denoted with t', is the best one among the preferences associated by def to any completion of t', t, to an assignment to con. The notation $t \downarrow_{con'}$ indicates the subtuple of t on the variables of con'.

A *solution* of a bipolar CSP $\langle S, V, C\rangle$ is a complete assignment to all variables in V, say s. Its overall preference is $ovpref(s) = ovpref_p(s) \times ovpref_n(s) = (p_1 \times_p \ldots \times_p p_k) \times (n_1 \times_n \ldots \times_n n_l)$, where, for $i := 1, \ldots, k$, $p_i \in P$, for $j := 1, \ldots, l$, $n_j \in N$, and $\exists\langle def_i, con_i\rangle \in C$ s.t. $p_i = def_i(s \downarrow_{con_i})$ and $\exists\langle def_j, con_j\rangle \in C$ s.t. $n_j = def(s \downarrow_{con_j})$. This is obtained by combining all the positive preferences associated to its subtuples on one side, all the negative preferences associated to its subtuples on the other side, and then compensating the two preferences so obtained. This definition is in accordance with cumulative prospect theory [10] used in bipolar decision making. A solution s is *optimal* if there is no other solution s' with $ovpref(s') > ovpref(s)$. Given a bipolar constraint $c = \langle def, con\rangle$ and one of its tuple t, it is possible to define two functions pos and neg as follows: $pos(c)(t) = def(t)$ if $def(t) \in P$, otherwise $pos(c)(t) = \Box$, and $neg(c)(t) = def(t)$ if $def(t) \in N$, otherwise $neg(c)(t) = \Box$.

3 Uncertain bipolar problems

Uncertain bipolar problems (UBCSPs) are characterized by a set of variables, which can be controllable or uncontrollable, and by a set of bipolar constraints. Thus, a UBCSP is a BCSP where some of the variables are uncontrollable. Moreover, the domain of every uncontrollable variable is equipped with a possibility distribution, that specifies, for every value in the domain, the degree of plausibility that the variable takes that value. Formally, a possibility distribution π associated to a variable z with domain A_Z is a mapping from A_Z to a totally ordered scale L (usually $[0,1]$) s.t. $\forall a \in A_Z$, $\pi(a) \in L$ and $\exists\ a \in A_Z$ s.t. $\pi(a) = \mathbf{1}$, where $\mathbf{1}$ the top element of the scale L [11].

Definition 1 (UBCSP). *An uncertain bipolar CSP is a tuple $\langle S, V_c, V_u, C_c, C_{cu}\rangle$, where*

- $S = (N, P, +, \times, \bot, \square, \top)$ *is a bipolar preference structure and $\leq_S$ is the ordering induced by operator $+$;*
- $V_c = \{x_1, \ldots x_n\}$ *is a set of controllable variables;*
- $V_u = \{z_1, \ldots z_k\}$ *is a set of uncontrollable variables, where every $z_i \in V_u$ has possibility distribution π_i with scale $[0,1]$;*
- C_c *is the set of bipolar constraints that involve only variables of V_c*
- C_{cu} *is a set of bipolar constraints that involve at least a variable in V_c and a variable in V_u and that may involve any other variable of $(V_c \cup V_u)$.*

In a BCSP, a solution is an assignment to all its variables. In a UBCSP, instead, a solution is an assignment to all its controllable variables.

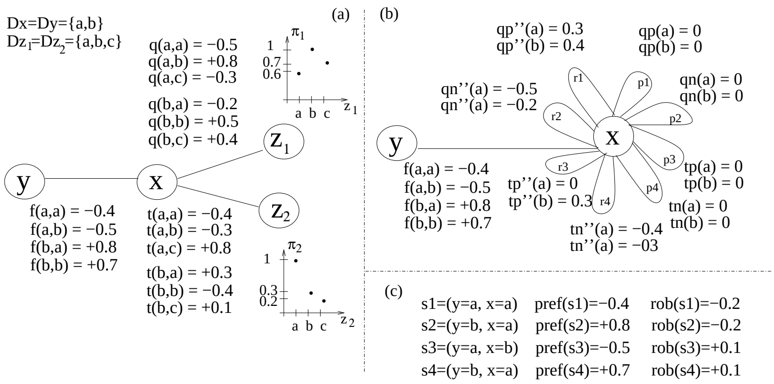

Fig. 1. How to handle a UBCSP.

An example of a UBCSP is presented in Figure 1 (a). It is defined by the tuple $\langle S, V_c = \{x, y\}, V_u = \{z_1, z_2\}, C_c, C_{cu}\}\rangle$, where S is the bipolar structure considered before, i.e., $\langle[-1, 0], [0, 1], max, \times, -1, 0, \mathbf{1}\rangle$, where $\times$ is s.t. $\times_p = max$, $\times_n = min$ and $\times_{np} = sum$. The set of controllable variables is composed by x and y, while the set of uncontrollable variables is composed by z_1 and z_2, which are characterized by the possibility distributions π_1 and π_2. The set of constraints C_c contains $\langle f, \{x, y\}\rangle$, while C_{cu} contains $\langle q, \{x, z_1\}\rangle$ and $\langle t, \{x, z_2\}\rangle$. Figure 1 (a) shows the positive and the negative preferences within such constraints and the possibility distributions π_1 and π_2 over the domains of z_1 and z_2. Other parts of Figure 1 will be described later.

4 Preference, robustness, and desirable properties

Given a solution s of a UBCSP, we will associate a preference degree to it, written $pref(s)$, which summarizes all the preferences in the controllable part and that can be obtained for some assignment to the uncontrollable variables decided by the Nature. It is reasonable to assume that $pref(s)$ belongs to the set of preferences in the considered bipolar preference structure.

When we deal with UBCSPs, we have to consider another interesting aspect that characterizes a solution, that is, its robustness with respect to the uncertainty, which measures what is the impact of Nature on the preference obtained by choosing that solution. The robustness of s will depend both on the preferences in the constraints connecting both controllable and uncontrollable variables to s and on such possibility distributions and it is also reasonable that it will be an element of the bipolar preference structure. This will allow us to use the operators of such a structure over the robustness values. Before giving our definition of robustness of a solution s, that we will denote with $rob(s)$, we define two properties that such a definition should satisfy as in [6, 9]. The first one states that, if we increase the preferences of any tuple involving uncontrollable variables, solution should have a higher value of robustness, the second one states that the same result should hold if we lower the possibility of any value of the uncontrollable variables.

Property 1 *Given solutions s and s' of a UBCSP, $\langle S, V_c, V_u, C_c, C_{cu}\rangle$, where every v_i in V_u is associated to a possibility distribution π_i, if $\forall$ $\langle def, con\rangle \in C_{cu}$ and $\forall a$ assignment to the uncontrollable variables in con, $def((s, a) \downarrow_{con}) \leq_S def((s', a) \downarrow_{con})$, then it should be that $rob(s) \leq_S rob(s')$.*

Property 2 *Given a solution s of a UBCSP $Q_i = \langle S, V_c, V_u, C_c, C_{cu}\rangle$. Assume variables in V_u are described by a possibility distribution π_i, for $i = 1, 2$ s.t. $\forall a$ assignment to variables in V_u, $\pi_2(a) \leq \pi_1(a)$. Then it should be that $rob_{\pi_1}(s) \leq_S rob_{\pi_2}(s)$, where rob_{π_i} is the robustness computed in the problem with possibility distribution π_i.*

To understand which solutions are better than others in a UBCSP, it is reasonable to consider a solution ordering, which should be reflexive and transitive. The notions of robustness and preference should be related to this solution ordering, say $\succ$, by the following properties 3, 4, and 5. Properties 3 and 4 state that two solutions which are equally good with respect to one aspect (robustness or preference degree) and differ on the other should be ordered according to the discriminating aspect. Property 5 states that, if two solutions s and s' are s.t. the overall preference of the assignment (s, a) to all the variables is better than the one of (s', a), $\forall a$ assignment to the uncontrollable variables, then s should be better than the other one.

Property 3 *Given two solutions s and s' of a UBCSP, if $rob(s) = rob(s')$ and $pref(s) >_S pref(s')$, then it should be that $s \succ s'$.*

Property 4 *Given two solutions s and s' of a UBCSP s.t. $pref(s) = pref(s')$, and $rob(s) >_S rob(s')$, then it should be that $s \succ s'$.*

Property 5 *Given two solutions s and s', a UBCSP $Q = \langle S, V_c, V_u, C_c, C_{cu}\rangle$, s.t. $ovpref(s, a) >_S ovpref(s', a)$, $\forall a$ assignment to V_u, then it should be that $s \succ s'$.*

5 Removing uncertainty: preference, robustness and semantics

We now propose a procedure that extends to a bipolar context a common approach used to deal with uncertainty, that eliminates uncontrollable variables preserving as much information as possible [6, 9]. Starting from this procedure, we define the robustness and preference degrees that satisfy the desirable properties mentioned above.

The procedure, that we call Algorithm *B-SP*, generalizes to the case of positive and negative totally ordered preferences defined over intervals of $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ (or structures isomorphic to them), Algorithm *SP* [9] for handling problems with fuzzy preferences and uncontrollable variables associated to possibility distributions. The algorithm takes as input a UBCSP $Q = \langle S, V_c, V_u, C_c, C_{cu} \rangle$, where every variable $z_i \in V_u$ has a possibility distribution π_i and where $S = \langle N,\, P,\, +,\, \times,\, \bot,\, \Box,\, \top \rangle$ is any bipolar preference structure with N and P totally ordered intervals of $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ (or structures isomorph to them). Then, the algorithm translates the UBCSP Q in the RBCSP $Q' = \langle S, V_c, C_c \cup C_{proj}, C_{rob} \rangle$ (that is, in the BCSP $\langle S, V, C \cup C_{rob} \rangle$). This problem Q' is obtained from Q by eliminating its uncontrollable variables and the bipolar constraints in C_{cu} relating controllable and uncontrollable variables, and by adding new bipolar constraints only among these controllable variables. These new constraints can be classified in two sets, that we call C_{rob} (robustness constraints) and C_{proj} (projection constraints), that we describe in the following. Starting from this problem Q', we then define the preference degree (resp., the robustness degree) of a solution considering the preference functions of the constraints in $C_c \cup C_{proj}$ (resp., in C_{rob}).

The set of *robustness constraints* C_{rob} is composed by the constraints obtained by reasoning on preference functions of the constraints in C_{cu} and on the possibilities associated to values in the domains of uncontrollable variables involved in such constraints. C_{rob} is built in three steps.

In the first step, that we denote *normalization*, every constraint $c = \langle def, con \rangle$ in C_{cu} s.t. $con \cap V_c = X$ and $con \cap V_u = Z$, is translated in two bipolar constraints $\langle defp, con \rangle$ and $\langle defn, con \rangle$, with preferences in $[0, 1]$, where, $\forall (t_X, t_Z)$ assignment to $X \times Z$, $defp(t_X, t_Z) = g_p(pos(c)(t_X, t_Z))$ and $defn(t_X, t_Z) = g_n(neg(c)(t_X, t_Z))$. If the positive (resp., negative) preferences are defined in the interval of $\mathbb{R}$ (or $\mathbb{Q}, \mathbb{Z}$), $P = [a_p, b_p]$ (resp., $N = [a_n, b_n]$) then g_p: $[a_p, b_p] \rightarrow [0, 1]$ (resp., g_n: $[a_n, b_n] \rightarrow [0, 1]$) is s.t. $x \mapsto \frac{x - a_p}{b_p - a_p}$ (resp., $x \mapsto \frac{x - a_n}{b_n - a_n}$) by using the classical division and subtraction operation of $\mathbb{R}$.

In the second step, denoted *uncontrollability elimination*, the constraint $\langle defp, con \rangle$ (resp., $\langle defn,\, con \rangle$) obtained before is translated in $\langle defp',\, X \rangle$ (resp., $\langle defn',\, X \rangle$), where, $\forall t_X$ assignment to X, $defp'(t_X) = \inf_{t_Z \in A_Z} \sup(defp(t_X, t_Z), c_S(\pi_Z(t_Z)))$, and $defn'(t_X) = \inf_{t_Z \in A_Z} \sup(defn(t_X,\, t_Z),\, c_S(\pi_Z(t_Z)))$, where c_S is an order reversing map w.r.t. $\leq_S$ in $[0, 1]$, s.t. $c_S(c_S(p)) = p$ and $\inf$, which is the opposite of the $\sup$ operator (derived from operator $+$ of S), applied to a set of preferences, returns its worst preference w.r.t. the ordering $\leq_S$.

In the third step, denoted *denormalization*, the constraint $\langle defp', X \rangle$ (resp., $\langle defn', X \rangle$) is translated in $\langle defp'', X \rangle$ (resp., $\langle defn'', X \rangle$), where $\forall t_X$ assignment to X, $defp''(t_X) = g_p^{-1}(defp'(t_X))$, and $defn''(t_X) = g_n^{-1}(defn'(t_X))$. The map g_p^{-1}:$[0, 1] \rightarrow [a_p, b_p]$ is s.t. $y \mapsto [y(b_p - a_p) + a_p]$, and g_n^{-1}:$[0, 1] \rightarrow [a_n, b_n]$ is s.t. $y \mapsto$

$[y(b_n - a_n) + a_n]$. Summarizing, given $c = \langle def, X \cup Z \rangle \in C_{cu}$, its corresponding robustness constraints in C_{rob} are the constraints $\langle defp'', X \rangle$ and $\langle defn'', X \rangle$ above.

Projection constraints are added to the problem in order to recall part of the information contained in the constraints in C_{cu} that will be removed later. They are useful to guarantee that the preference degree of a solution, say $pref(s)$, that we will define later, is a value that can be obtained in the given UBCSP. The set of projection constraints C_{proj} is defined as follows. Given a bipolar constraint $c = \langle def, con \rangle$ in C_{cu}, s.t. $con \cap V_c = X$ and $con \cap V_u = Z$, then the corresponding bipolar constraints in C_{proj} are $\langle defp, X \rangle$ and $\langle defn, X \rangle$, where $defp(t_X) = \inf_{\{t_Z \in A_Z\}} pos(c)\ (t_X, t_Z)$ and $defn(t_X) = \sup_{\{a \in A_Z\}} neg(c)\ (t_X, t_Z)$. in other words, $defn(t_X)$ (resp., $defp(t_X)$) is the best negative (resp., the worst positive) preference that can be reached for t_X in c for the various values t_Z in the domain of the uncontrollable variables in Z.

Let us show via an example how *B-SP* works. Consider the UBCSP $Q = \langle S, V_c = \{x, y\}, V_u = \{z_1, z_2\}, C_c, C_{cu} \rangle$ in Figure 1 (a). Figure 1 (b) shows the RBCSP $Q' = \langle S, V_c = \{x, y\}, C_c \cup C_{proj}, C_{rob} \rangle$, built by algorithm *B-SP*. C_c is composed by $\langle f, \{x, y\} \rangle$. C_{proj} is composed by $p1 = \langle qp, \{x\} \rangle$, $p2 = \langle qn, \{x\} \rangle$, $p3 = \langle tp, \{x\} \rangle$ and $p4 = \langle tn, \{x\} \rangle$, while C_{rob} by $r1 = \langle qp'', \{x\} \rangle$, $r2 = \langle qn'', \{x\} \rangle$, $r3 = \langle tp'', \{x\} \rangle$ and $r4 = \langle tn'', \{x\} \rangle$. Constraints in C_{rob} are obtained by assuming g_p the identity map, and $g_n : [-1, 0] \to [0, 1]$ s.t. $n \mapsto n + 1$.

Starting from the RBCSP $Q' = \langle S, V_c, C_c \cup C_{proj}, C_{rob} \rangle$, obtained applying algorithm *B-SP* to the BCSP Q, we associate to each solution of Q, a pair composed by a degree of preference and a degree of robustness. The preference of a solution is obtained by compensating a positive and a negative preference, where the positive (resp., the negative) preference is obtained by combining all positive (resp., negative) preferences of the appropriate subtuples of the solution over the constraints in $C_c \cup C_{proj}$, i.e., over initial constraints of Q linking only controllable variables and over new projection constraints. The robustness is obtained similarly, but considering only the constraints in C_{rob}, i.e., in the robustness constraints. It is possible to prove that this definition of robustness satisfies Properties 1 and 2.

Definition 2 (preference and robustness). *Given a solution s of a UBCSP Q, let $Q' = \langle S, V_c, C_c \cup C_{proj}, C_{rob} \rangle$ the RBCSP obtained from Q by algorithm* B-SP. *The* preference *of s is $pref(s) = pref_p(s) \times pref_n(s)$, where $\times$ is the compensation operator of S, $pref_p(s) = \Pi_{\{\langle def, con \rangle \in C_c \cup C_{proj}\}}\ pos(c)(s \downarrow_{con})$, $pref_n(s) = \Pi_{\{\langle def, con \rangle \in C_c \cup C_{proj}\}}\ neg(c)(s \downarrow_{con})$. The* robustness *of s is $rob(s) = rob_p(s) \times rob_n(s)$, where $rob_p(s) = \Pi_{\{\langle def, con \rangle \in C_{rob}\}}\ pos(c)(s \downarrow_{con})$, $rob_n(s) = \Pi_{\{\langle def, con \rangle \in C_{rob}\}}\ neg(c)(s \downarrow_{con})$.*

A solution of a BCSP is associated to a preference and a robustness degree. We here define *semantics* to order the solutions which depend on our attitude w.r.t. these two notions. Assume to have $A1 = (pref_1, rob_1)$ and $A2 = (pref_2, rob_2)$. The first semantics, which is called *Risky*, states that $A1 \succ_{Risky} A2$ iff $pref_1 >_S pref_2$ or ($pref_1 = pref_2$ and $rob_1 >_S rob_2$). The idea is to give more relevance to the preference degree. The second semantics, called *Safe*, states that $A1 \succ_{Safe} A2$ iff $rob_1 >_S rob_2$ or ($rob_1 =_S rob_2$ and $pref_1 >_S pref_2$). It represents the opposite attitude w.r.t. Risky semantics, since it considers the robustness degree as the most important feature. The last semantics, called *Diplomatic*, aims at giving the same importance to preference and robustness. $A1 \succ_{Dipl} A2$ iff ($pref_1 \geq_S pref_2$ and $rob_1 \geq_S rob_2$) and ($pref_1 >_S pref_2$ or $rob_1 >_S rob_2$). By definition, the Risky, Safe and Diplomatic semantics satisfy Properties 3 and 4. Additionally, Risky satisfies Property 5, if $\times$ in the bipolar structure is strictly monotonic. Property 3, 4 and 5 are desirable. However, there are semantics that don't satisfy them. For example, this happens with a semantics, that we denote *Mixed*, that generalizes the one adopted in [6]: $A1 \succ_{Mixed} A2$ iff $pref_1 \times rob_1 >_S pref_2 \times rob_2$, where $\times$ is the compensation operator in the bipolar structure.

6 Conclusions

We have studied problems with bipolar preferences and uncontrollable variables with a possibility distribution over such variables. Our technical development, although being an extension of two previous lines of work, which dealt with bipolarity only, or only uncertainty, was not strighforward, since it was not clear if it was possible to deal simultaneously with possibilistic uncertainty and bipolar preferences, making sure that desirable properties hold. In fact, such a task could have required a bipolarization of the possibility scale. Our results instead show that it is possible, without any added bipolarization, to extend the formalism in [3] to bipolar preferences and the one in [9] to uncertainty, preserving the desired properties.

Acknowledgements

This work has been supported by Italian MIUR PRIN project "Constraints and Preferences" (n. 2005015491).

References

1. S. Benferhat, D. Dubois, S. Kaci, and H. Prade. Bipolar possibility theory in preference modeling: representation, fusion and optimal solutions. *Information Fusion*, 7(1), 2006.
2. S. Bistarelli, U. Montanari, and F. Rossi. Semiring-based constraint solving and optimization. *Journal of the ACM*, 44(2):201–236, mar 1997.
3. S. Bistarelli, M. S. Pini, F. Rossi, and K. B. Venable. Bipolar preference problems: framework, properties and solving techniques. In *Recent Advances in Constraints. Selected papers from 2006 ERCIM workshop on constraints*. Springer LNAI, 2007.
4. D. Dubois and H. Fargier. On the qualitative comparison of sets of positive and negative affects. In *ECSQARU'05*, pages 305–316, 2005.
5. D. Dubois and H. Fargier. Qualitative decision making with bipolar information. In *KR'06*, pages 175–186, 2006.
6. D. Dubois, H. Fargier, and H. Prade. Possibility theory in constraint satisfaction problems: Handling priority, preference and uncertainty. *Appl. Intell.*, 6(4):287–309, 1996.
7. H. Fargier, J. Lang, R. Martin-Clouaire, and T. Schiex. A constraint satisfaction framework for decision under uncertainty. In *UAI-95*, pages 167–174. Morgan Kaufmann, 1995.
8. M. Grabisch and Ch. Labreuche. Bi-capacities - parts i and ii. *Fuzzy Sets and Systems*, 151:211–260, 2005.
9. M. S. Pini, F. Rossi, and K. B. Venable. Possibility theory for reasoning about uncertain soft constraints. In *ECSQARU-05*, volume 3571 of *LNCS*, pages 800–811. Springer, 2005.
10. A. Tversky and D. Kahneman. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5:297–323, 1992.
11. L.A. Zadeh. Fuzzy sets as a basis for the theory of possibility. *Fuzzy sets and systems*, pages 13–28, 1978.